Department of Physics
PRELIMINARY EXAMINATION 2018
Part I. Short Questions/Answers

Thursday May 17th, 2018, 14-17h

Examiners: Prof. J. Cline, Prof. K. Dasgupta, Prof. H. Guo, Prof. G. Gervais (Chair), Prof. D. Hanna, and Prof. V. Kaspi.

INSTRUCTIONS

Answer **10** *questions out of the choice of 16.*

This is a **closed book** exam. Approved calculators may be used (non-programmable ones), though approximate numerical results are valid.

If you attempt more than ten questions, you should clearly mark which ones should be graded.

DO NOT WRITE YOUR NAME; write **ONLY** your student ID on the exam booklet. Clearly indicate the question number next to each answer.

This exam has 8 pages, including this title page.

1. **How to measure in quantum mechanics...**

   Operator $\hat{A}$ of an observable α has two normalized eigenfunctions ϕ_1 and ϕ_2 with eigenvalues α_1 and α_2 respectively. Another operator $\hat{B}$ of observable β has normalized eigenfunctions χ_1 and χ_2, with eigenvalues β_1 and β_2. The eigenfunctions are somehow related by the following expressions:

   $$\phi_1 = \frac{2\chi_1 + 3\chi_2}{\sqrt{13}}; \qquad \phi_2 = \frac{3\chi_1 - 2\chi_2}{\sqrt{13}}.$$

   A measurement is made on α and the value α_1 is obtained. Afterward β is measured followed by measuring α again, what is the probability of obtaining α_1 after the final measurement?

2. **The cylindrical 'planet'**

   Imagine a planet being a long infinite solid cylinder of radius R with a mass per unit length Λ. The matter is uniformly distributed over its radius. Find the potential and gravitational field everywhere, *i.e.* inside and outside the cylinder, and sketch the field lines.

3. **A classical pencil!**

   A pencil of mass m and length ℓ is balanced on its tip, initially with a very small angle θ_0 from the vertical, and a very small initial angular velocity $\dot{\theta}_0$. The motion is in a plane so only one angle is needed. Write down and solve the linearized equation of motion that describes the small-angle motion. From this, make an order of magnitude estimate for the time scale for the pencil to fall down. Finally, maximize this time scale subject to the constraint that $I\dot{\theta}_0\theta_0 = \hbar/2$, where I is the moment of inertia. (Even though this is a classical problem, these initial conditions correspond to the minimum values allowed by quantum mechanics.)

4. **Lagrange meets Einstein**

   Starting from the relativistic Lagrangian $L = -mc^2\sqrt{1-\beta^2} - V$ ($\beta \equiv v/c$ with c the speed of light) for a particle of mass m moving at relativistic velocity v in a potential V, calculate Hamilton's function for that same particle. Taking the non-relativistic limit, discuss the meaning of the extra term you obtain in addition to $T + V$. Does this extra term affect the equations of motion?

5. **Absolutely ideal**

   Helium gas is known as being very closely approximated as an ideal (spinless) gas. Imagine that you are provided with helium gas contained in a box that has a very small aperture with a diameter much smaller that the average mean free path between two helium atoms. The pressure inside the box can be varied, keeping the average mean free path always larger than the aperture diameter. Using a detector that can count the particles coming out and measure their kinetic energies, provide the probability distribution you would measure (as a function the velocities of the particles) and make a sketch. If the temperature is hypothetically decreased towards zero (with the helium retaining its gas form), will this distribution always be the same?

6. **A quantum pencil (!!!)**

   Write the Hamiltonian for a pencil of mass m, length ℓ and moment of inertia I that is free to rotate around its tip under the influence of gravity, and expand the potential to lowest order for small angles θ from the vertical. You can assume the rotation is just in the θ direction, ignoring the azimuthal angle. Write the corresponding Schrödinger equation, ignoring the constant in the potential, and show that the solutions take the form

$$\psi = Ae^{-ia\theta^2/2+bt} + Be^{ia\theta^2/2-bt}$$

   Choose the linear combination that best localizes ψ around $\theta = 0$ initially, at $t = 0$. Determine the order of magnitude of $\Delta\theta$ at $t = 0$, and the time scale for developing the other linear combination for which θ is no longer localized near zero.

7. **Relativistic reaction**

   Consider the reaction $\gamma + p \rightarrow \pi^0 + p$ in a frame where the target proton is at rest and the gamma ray travels along the x axis.

   (a) Find the minimum energy the gamma ray must have in this frame to initiate this reaction. Use $m_p c^2 = 940\ MeV$ and $m_\pi c^2 = 140\ MeV$.

   (b) Calculate the three-momentum of the π^0 in this frame.

8. **A battle between Heisenberg and hydrogen**

   In an eigenstate the uncertainty in energy is exactly zero, so the uncertainty in time is infinite. This means an electron in an eigenstate will stay there forever. How is this compatible with the fact that, in a hydrogen atom, an electron may jump from one eigenstate to another giving rise to the spectra that we can observe? Does this violate the uncertainty principle?

9. **Sun's blackbody**

   (a) Consider the sun to be a blackbody with temperature T_s and radius r_s; similarly for Mercury with temperature T_m and radius r_m. Take the distance from Mercury to the sun to be R. Find the temperature of Mercury assuming it gets all its heat from the sun and is in a steady state.

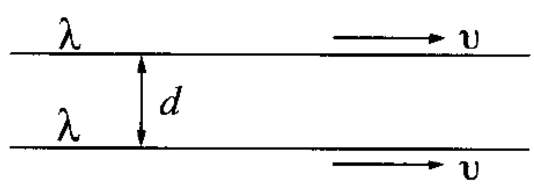
   (b) Suppose at some moment the sun runs out of nuclear fuel, so there is no more source of energy to maintain it at constant temperature. Take its heat capacity to be C. Derive a differential equation describing how fast the temperature decreases and solve it in terms of the initial temperature T_0.

10. **The physics of a mirror**

    Consider an interface separating air with and index of refraction $n = 1$ and a metallic plate with a free electron density $\mathbf{j}$. An electromagnetic wave of angular frequency ω and wave vector $\mathbf{k}$ travels towards the metallic plate at normal incidence, *i.e.* with its wave vector perpendicular to the metallic plate. Write down the Maxwell's equations on both sides of the interface, and state the boundary conditions at the interface for the plane wave solutions. Calculate the dispersion relation everywhere, and provide a sketch of what is happening to the wave amplitude at the interface, and inside the metallic plate.

11. **Charges racing on two parallel lines**

    Consider two line charges, parallel and separated by a distance d. Each has linear charge density λ and they are moving in the same direction with velocity v, as shown in the figure.



(a) What is the electrostatic force between the two?

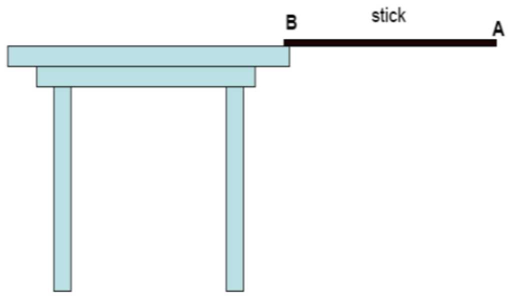
(b) For what value of v is the force in (a) balanced by the magnetic force between the two lines?

12. **Stick on the table**

    A uniform stick of mass M and length L is held horizontally with its end B on the edge of a table, and the other end A by hand, see figure below.

    (a) Calculate the moment of inertia around the centre of mass of the stick. What is the moment of inertia around point B?

    Now, point A is suddenly released. At the instant just after release:

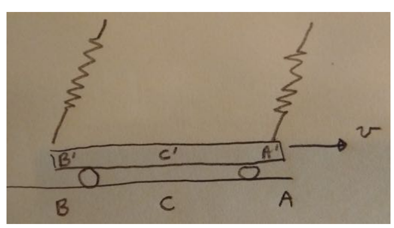


(b) What is the torque about B?

(c) What is the angular acceleration about B? What is the vertical acceleration of the centre of mass?

13. **Train and several folks looking at a lightning**

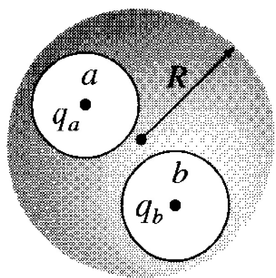
A railway car of length L_0 moves past a station with velocity v. There are observers A', B' and C' located at the front, back and middle of the car, and corresponding observers located on the platform at positions A, B and C. See figure below.



The railway car and platform are struck by lightning at the front and back of the car. The lightning bolts are simultaneous in the reference frame of the platform. An observer at C, halfway between A and B, on the platform, sees the lightning bolts at A and B as being simultaneous. What does an observer on the train at C' see?

14. **The outcome of making 'bubbles' in a conductor**

Two spherical cavities, of radius r_a and r_b, are hollowed out from the interior of a neutral spherical conductor of radius R, as shown in the figure.



Point charges q_a and q_b are placed at the centres of the cavities.

(a) Find the surface charge densities σ_a, σ_b, and σ_R.

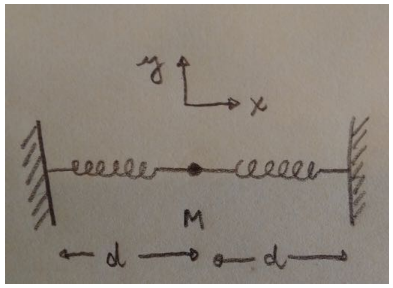
(b) What is the electrostatic field outside the conducting sphere?

(c) What is the field inside each cavity?

(d) What is the force on q_a?

15. **Competing planar oscillations**

An object of mass M rests on a frictionless horizontal surface. Two identical springs of spring constant k and relaxed length l_0 are attached to M, see figure below. The object is at rest in static equilibrium when each spring is of length d (where $d > l_0$). How does the frequency of small oscillations in the x direction compare to that in the y direction?



16. **'Relative' hearing**

A speaker at the front of a room and an identical speaker at the rear of the room are being driven by the same oscillator, with a frequency of 525 Hz. A student walks at a uniform speed of 1.5 m/s along the line between the two speakers. The speed of sound in the room is 350 m/s and its length is 20 m.

(a) How many beats per second does the student hear?

(b) What is the distance between regions of maximum intensity along the line between the speakers?